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Mortgage Refinancing and Consumption Smoothing

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Mortgage refinancing and consumption smoothing^{*}

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Abstract

This paper analyses the optimal refinancing decision of an agent whose only asset in the portfolio is the house where she lives in the context of a life-cycle model. The mortgage is modelled as an adjustable rate contract covering the remaining life of the house owner. Thus, refinancing concerns only the size of the mortgage, which can be adjusted in any period subject to a constraint on the amount that can be borrowed: the value of the new mortgage cannot exceed the latest realised price. The paper solves the model analytically and then numerically calibrates the refinancing decision.

Keywords: mortgage refinancing, consumption smoothing.

JEL Classifications: D11, D91, G21

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1 Introduction

Home equity is the most widely held asset and, therefore, an important component of household wealth in many countries. In the U.K., the typical consumer is a homeowner and the majority of her wealth is locked into the house. The role of housing is made more complicated by the fact that it serves a double purpose: it is both an investment vehicle that allows investors to hold home equity and a durable consumption good from which the owner derives utility. In addition, moving house involves high transaction costs and this makes the trading infrequent. However, home equity may still act as a buffer against bad income shocks by serving as collateral for secured loans and through remortgaging, borrowers can alter their debt position without moving their properties. Figure 1 shows that in recent years household borrowing secured against housing has risen by a considerably greater amount than that needed to fund new housing investments¹.

The equity that consumers release from the value of their home may be used to invest in other assets in order to rebalance the portfolio (*financial motivation*) or to finance consumer spending (*consumption smoothing motivation*). A recent stream of the literature has shown that the reasons why investors may choose to refinance and the extent to which they do so largely depend on the amount of liquid assets that they hold. Hurst and Stafford (2004), using micro data from the Panel Study of Income Dynamics for the US, find that households who experience a spell of unemployment

¹The Bank of England's estimate of mortgage equity withdrawal (MEW) measures that part of consumer borrowing from mortgage lenders that is not invested in the housing market. MEW takes the income in housing finance (net mortgage lending and capital grants) and subtracts households' investments in housing (purchases of new houses and houses from other sectors, improvements to property and the transaction costs of moving house).

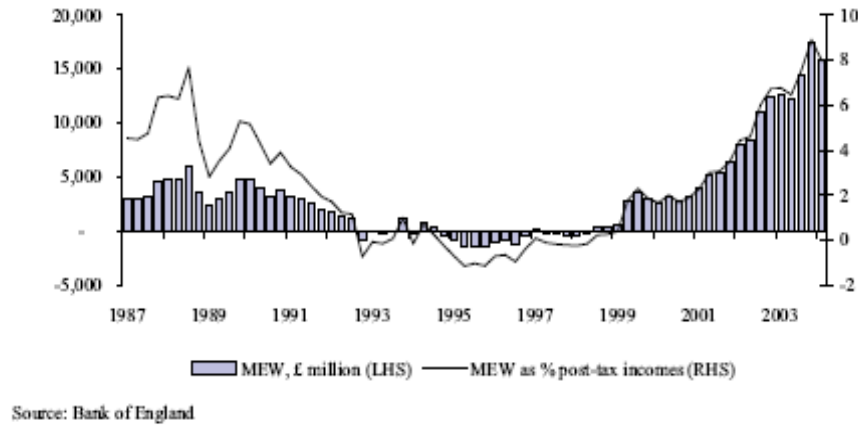


Figure 1: Bank of England's estimate of mortgage equity withdrawal

and have zero liquid assets are 25% more likely to refinance. Their empirical evidence seems to suggest that, on average, such households convert most of the equity that they remove through refinancing into current consumption. On the contrary, non-liquidity constrained households refinance driven by financial reasons and reallocate the equity to other portfolio components. This result is partly consistent with the findings of Vass and Smith (2004) that, using data from the Survey of English Housing, show that for low-income households the amount of equity withdrawn is proportionately larger relative to income than for higher income groups.

This paper analyses the optimal refinancing decision of an agent whose only asset in the portfolio is the house where she lives in the context of a life-cycle model: the investor can borrow only through the mortgage (i.e. using the house as collateral) and saving is precluded by assumption. The no saving hypothesis might seem difficult to justify; however, data from the Family Resources Survey show that there is a large number of households who are homeowners but whose savings are negligible [INSERT

TABLE]. Furthermore, this assumption is needed to abstract from any portfolio considerations and to focus only on the use of mortgage refinancing as a mechanism by which individuals smooth consumption over time. Since in the model there are no other assets but the house, the only reason why investors remortgage is to smooth consumption. The mortgage is modelled as an adjustable rate contract covering the remaining life of the house owner. Thus, refinancing concerns only the size of the mortgage, which can be adjusted in any period subject to a constraint on the amount that can be borrowed: the value of the new mortgage cannot exceed the latest realised price. The refinancing may either involve increasing (mortgage equity withdrawal) or reducing (mortgage equity injection) the outstanding debt. Under the assumption of no uncertainty, I derive an analytical solution to the problem and then I numerically calibrate the refinancing decision. Though there are costs to obtaining the closed-form solution, such as the no savings assumption, the benefit is that the solution is easy to interpret and I gain several economic insights. The theoretical results are consistent with the empirical evidence of Hurst and Stafford (2004) for the US and Smith and Vass (2004) for the UK: when current income is low with respect to future income, consumers with no financial assets in their portfolios respond by releasing equity from their house and, by doing so, they smooth consumption over time.

The paper is organised as follows. Section 2 lays out the model in the case in which refinancing the mortgage is costless. Section 3 derives the optimal refinancing policy. Section 4 extends the model by introducing a fixed transaction cost that has to be paid to adjust the mortgage size. Section 5 presents calibrated simulations. Section 6 concludes.

2 The model assumptions

To explore the use of home equity as a mechanism by which individuals smooth their consumption over time, I assume that households can borrow only through the mortgage and the only asset in their portfolio is the house where they live. Consumers live for a finite period of time and the length of life (T) is known in advance. In every period before the final one ($t < T$), the budget constraint is:

$$c_t = w_t + (1 - \rho_t)M_{t+1} - M_t \quad (1)$$

where c_t is consumption of period t , w_t is labour income, M_t is the mortgage debt and ρ_t the mortgage rate, which is time-varying but perfectly foreseen.

Since borrowings and savings in a financial asset are precluded by assumption, individuals maximise life-time utility through their mortgage decisions: in each period they can costlessly redeem their existing mortgage (M_t) and take out a new loan with maturity date T (M_{t+1}), where $M_{t+1} = M_t$ if no refinancing is undertaken². The only source of imperfection in the mortgage market is that the new loan cannot exceed the latest realised price p_t .

For tractability, I assume that lifetime preferences are additive and the instantaneous utility function is concave and depends only on current consumption:

$$U_0 = \sum_{t=0}^T \phi^t u(c_t)$$

²Therefore, when the optimal mortgage is equal to the current mortgage, consumers have to pay off their existing debt and take out a new loan of the same size. This is without loss of generality since refinancing is costless.

where $\phi > 0$ is the rate of time preference. By excluding labour income and housing from the utility function, I implicitly assume that housing is homogeneous (agents occupy a house of a fixed size and quality throughout their life) and that there is an inelastic labour supply. In addition, I do not include any form of uncertainty; thus, the mortgage rate ρ_t , labour income w_t and house prices p_t may be time varying, but are perfectly foreseen.

In the last period (T), any outstanding mortgage is paid off and the house is sold at the price p_T . Since I abstract from bequests and inheritances, there is no issue of refinancing for period $T + 1$:

$$c_T = w_T - M_T + p_T$$

and the value function is

$$V_T = u(c_T) = u(w_T - M_T + p_T)$$

3 The optimal refinancing policy

At $T - 1$ the individual solves:

$$\begin{aligned} V_{T-1} &= \max_{M_T} \nu_{T-1} = \max_{M_T} (u(c_{T-1}) + \phi V_T) \\ &= \max_{M_T} u(w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1}) + \phi u(w_T - M_T + p_T) \end{aligned}$$

For periods before $T - 1$, the value function at t can be defined recursively as:

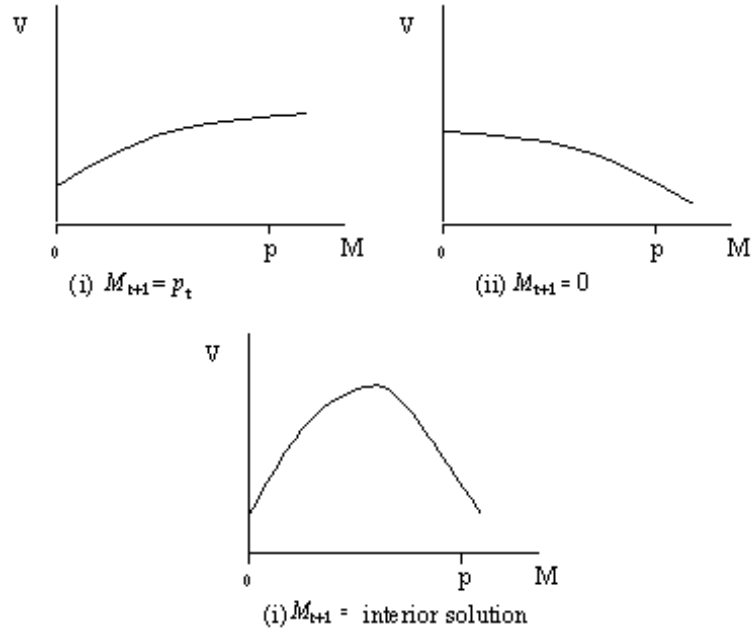


Figure 2: Optimal mortgage size

$$\begin{aligned}
 V_t &= \max_{M_{t+1}} \nu_t = \max_{M_{t+1}} (u(M_t, M_{t+1}) + \phi V_{t+1}) \\
 &= \max_{M_{t+1}} \left[u(M_t, M_{t+1}) + \phi \left\{ \max_{M_{t+2}} (u(M_{t+1}, M_{t+2}) + \phi V_{t+2}) \right\} \right]
 \end{aligned}$$

where $0 < M_{t+1} < p_t$. In each period consumers choose the optimal mortgage to carry into next period, given their labour income, the value of their house, the mortgage rate and their rate of time preference. As shown in figure 2, three cases arise, depending on the sign of the derivative of ν_t with respect to M_{t+1} . In particular, the consumer pays

off the debt if $\left. \frac{\partial \nu_t}{\partial M_{t+1}} \right|_{M_{t+1}=0} < 0$, she refinances to the maximum possible extent if $\left. \frac{\partial \nu_t}{\partial M_{t+1}} \right|_{M_{t+1}=p_t} > 0$ and she choose a mortgage size that solves $\frac{\partial \nu_t}{\partial M_{t+1}} = 0$ otherwise.

Proposition 1

$$\frac{\partial \nu_t}{\partial M_{t+1}} = u'(c_t)(1 - \rho_t) - \phi u'(c_{t+1}) \quad (2)$$

Proof. See the appendix. ■

Therefore, in any period before the final one the consumer chooses a maximum mortgage if the marginal utility of withdrawing equity from the house in the current period is higher than the future discounted marginal disutility of having to pay off the debt. At $t < T - 1$:

$$u'(w_t + (1 - \rho_t)p_t - M_t)(1 - \rho_t) > \phi u'(w_{t+1} + (1 - \rho_{t+1})M_{t+2} - p_t)$$

On the contrary, the individual pays off the loan if

$$u'(w_t - M_t)(1 - \rho_t) < \phi u'(w_{t+1} + (1 - \rho_{t+1})M_{t+2})$$

Otherwise there is an interior mortgage that solves

$$u'(w_t + (1 - \rho_t)M_{t+1} - M_t)(1 - \rho_t) = \phi u'(w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1})$$

Proposition 2 *If the consumer is "time neutral" ($\phi = 1 - \rho_t = 1 - \rho$), in any period*

before the final one the optimal refinancing policy is as follows. Define:

$$F_t = w_{t+1} - w_t + M_t + (1 - \rho)M_{t+2} \quad \text{for } t = 1 \dots T - 2 \quad (3)$$

$$= w_T - w_{T-1} + M_{T-1} - p_T \quad \text{for } t = T - 1 \quad (4)$$

Then:

$$M_{t+1} = \begin{cases} 0 & \text{if } F_t < 0 \\ p_t & \text{if } F_t > (2 - \rho)p_t \\ \frac{F_t}{2 - \rho} & \text{otherwise} \end{cases}$$

Proof. It follows directly from the fact that $u'(w_t + (1 - \rho)M_{t+1} - M_t) \leq u'(w_{t+1} + (1 - \rho)M_{t+2} - M_{t+1})$ implies $w_t + (1 - \rho)M_{t+1} - M_t \leq w_{t+1} + (1 - \rho)M_{t+2} - M_{t+1}$.

■

Proposition 2 states that the extent to which time-neutral consumers refinance depends positively on the wage differential: the lower the current wage relatively to the future wage, the higher the optimal size of the mortgage. This result is consistent with the empirical evidence of Hurst and Stafford (2004) for the US and of Smith and Vass (2004) for the UK: consumers with low current income and with no assets in their portfolios are those who release more equity from the house.

Corollary 2.1 *If wages are non-decreasing, the corner solution $M_t = 0$ is always suboptimal.*

Proof. Refer to (3). If $w_{t+1} \geq w_t$, $F_t > 0$ since M_t and M_{t+2} are bounded to be non-negative. ■

Corollary 2.2 *If $\phi = 1 - \rho_t$ and wages are constant, the optimal refinancing policy is given by*

$$\begin{aligned} M_{t+1} &= p_t && \text{if } F_t > (2 - \rho)p_t \\ M_{t+1} &= \frac{M_t + (1 - \rho)M_{t+2}}{2 - \rho} > 0 && \text{otherwise} \end{aligned}$$

That is, the mortgage at $t + 1$ is either p_t or a weighted average of the mortgage at t and the mortgage at $t + 2$. It should be noted that if the investor chooses an interior solution for $n \geq 3$ subsequent periods, the optimal mortgage solves a second order homogeneous difference equation with constant coefficients:

$$M_{t+2} - \frac{2 - \rho}{1 - \rho} M_{t+1} + \frac{M_t}{1 - \rho} = 0$$

The associated characteristic equation is:

$$m^2 - \frac{2 - \rho}{1 - \rho} m + \frac{1}{1 - \rho} = 0$$

Therefore, there are two distinct real roots:

$$m_{1,2} = \frac{1}{2} \left(\frac{2 - \rho}{1 - \rho} \pm \sqrt{\left(\frac{2 - \rho}{1 - \rho} \right)^2 - \frac{4}{1 - \rho}} \right) = \begin{matrix} \nearrow & 1 \\ \searrow & 1/(1 - \rho) \end{matrix}$$

and the general solution of the equation is:

$$M_t = A \left(\frac{1}{1 - \rho} \right)^t + B$$

where A and B are given by the initial conditions. Since the root m_2 is greater than 1, the equation is unstable and M_t will tend to decrease or grow exponentially over time depending on the sign of A . However, this is prevented by the upper and lower bounds on M_{t+1} ($0 \leq M_{t+1} \leq p_t$) and the constraint that at T the individual has to die without any debt.

Another important implication of proposition 1 is that when current labour income is low and investors respond by withdrawing equity, by doing so they smooth consumption over time. However, even if in the model there is no uncertainty, individuals might be prevented from smoothing consumption completely by the constraint on the amount of equity that can be withdrawn from the house: $0 \leq M_{t+1} \leq p_t$.

From the budget constraint (1), I derive equation (5), which links consumption in period t and $t + 1$ and thus it defines the evolution of consumption over the life-cycle:

$$\begin{aligned}\Delta c_{t+1} &= c_{t+1} - c_t = w_{t+1} - w_t + (1 - \rho)M_{t+2} - (2 - \rho)M_{t+1} + M_t \quad (5) \\ &= F_t - (2 - \rho)M_{t+1}\end{aligned}$$

where $\Delta c_{t+1} = 0$ only if there is an interior solution for the mortgage.

3.1 CARA utility function

To find a closed-form solution to the problem in the more general case in which the consumer is not necessarily time-neutral, one needs to specify the form of the utility

function. First, assume that the instantaneous utility function has a CARA form:

$$u(c_t) = 1 - \exp(-bc_t) \quad (6)$$

where b is the coefficient of absolute risk aversion. The value function at t is:

$$\begin{aligned} V_t &= \max \left[\sum_{s=t}^{T-1} \phi^{s-t} (1 - \exp(-b(w_s + (1 - \rho_s)M_{s+1} - M_s))) \right. \\ &\quad \left. + \phi^{T-t} (1 - \exp(-b(w_T - M_T + p_T))) \right] \\ &= \max v_t \end{aligned}$$

Proposition 3 *The optimal refinancing policy is as follows. Define:*

$$\begin{aligned} F_t^{CA} &= -\frac{1}{b} \ln \left(\frac{\phi}{1 - \rho_t} \right) + w_{t+1} - w_t + M_t + (1 - \rho_{t+1})M_{t+2} \text{ for } t = 1 \dots T-2 \\ &= -\frac{1}{b} \ln \left(\frac{\phi}{1 - \rho_{T-1}} \right) + w_T - w_{T-1} + M_{T-1} - p_T \text{ for } t = T-1 \end{aligned}$$

Then

$$M_{t+1} = \begin{cases} 0 & \text{if } F_t^{CA} < 0 \\ p_t & \text{if } F_t^{CA} > (2 - \rho_t)p_t \\ \frac{F_t^{CA}}{2 - \rho_t} & \text{otherwise} \end{cases}$$

Proof. From equation 2

$$\begin{aligned} \frac{\partial v_t}{\partial M_{t+1}} &= b(1 - \rho_t) \exp(-b(w_t + (1 - \rho_t)M_{t+1} - M_t)) \\ &\quad - \phi b \exp(-b(w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1})) \end{aligned}$$

and $M_{t+1} = 0$ when $\frac{\partial v_t}{\partial M_{t+1}}$ at 0 is negative, $M_{t+1} = p_t$ when $\frac{\partial v_t}{\partial M_{t+1}}$ at p_t is positive and it is an interior solution otherwise, where

$$\frac{\partial v_t}{\partial M_{t+1}} \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad \text{if} \quad \begin{matrix} (1 - \rho_t) \exp(-b(w_t + (1 - \rho_t)M_{t+1} - M_t)) \\ \phi \exp(-b(w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1})) \end{matrix} \begin{matrix} \leq \\ \geq \end{matrix}$$

Taking the logarithm of both sides gives

$$-\frac{1}{b} \ln \left(\frac{\phi}{1 - \rho_t} \right) + w_{t+1} - w_t + M_t + (1 - \rho_{t+1})M_{t+2} \begin{matrix} \leq \\ \geq \end{matrix} (2 - \rho_t)M_{t+1}$$

where the RHS is the function F_t^{CA} defined in the proposition ■

Proposition 5 states that the extent to which consumers refinance depends not only on the wage differential, as shown above, but also on their degree of “impatience” $\left(\frac{1-\rho_t}{\phi}\right)$. *Ceteris paribus*, the more impatient the consumer, the higher will be the optimal mortgage size. However, if the coefficient of risk aversion tends to zero, then:

$$\begin{aligned} \lim_{b \rightarrow 0} F_t^{CA} &= \infty \text{ if } 1 - \rho_t > \phi \\ \lim_{b \rightarrow 0} F_t^{CA} &= -\infty \text{ if } 1 - \rho_t < \phi \end{aligned}$$

This means that when $b \rightarrow 0$ an impatient consumer will take out a maximum mortgage, while a patient one will redeem any outstanding debt. Therefore, risk-neutral consumers always choose a corner solution and the optimal mortgage policy is bang-bang: M_{t+1} is either 0 or p_t depending on the individual degree of impatience.

Corollary 3.1 *If the investor chooses an interior solution for $n \geq 3$ subsequent periods, the optimal mortgage solves a second order non-homogeneous difference equation with non constant coefficients:*

$$M_{t+2} - \frac{2 - \rho_t}{1 - \rho_{t+1}} M_{t+1} + \frac{M_t}{1 - \rho_{t+1}} = \frac{1}{b} \ln \left(\frac{\phi}{1 - \rho_t} \right) - w_{t+1} + w_t$$

Since the time horizon is finite, it is possible to solve this equation backwards.

Corollary 3.2 *If wages are non-decreasing and $\phi \leq 1 - \rho_t$, the corner solution $M_t = 0$ is always suboptimal.*

Proof. Refer to the definition of F_t^{CA} . Since the mortgage is always non-negative, if $w_{t+1} \geq w_t$ and $\phi \leq 1 - \rho_t$, then $F_t^{CA} \geq 0$. ■

This means that if the wage today is lower than the wage tomorrow and if agents highly discount the future (they are "impatient"), it is never optimal to pay off the mortgage. This result is the extension of corollary 3 to an impatient consumer in the case of a CARA per-period felicity function.

CARA utility is often considered theoretically unattractive because it does not rule out negative consumption ($u'(0) > 0$). However, it is possible to prove that in this framework consumption is always non-negative, at least in the case of a time-neutral or impatient consumer.

Proposition 4 *For a time-neutral or impatient investor, consumption is always non-negative.*

Proof. Since c_t positively depends on M_{t+1} , to prove that consumption is always non-negative is equivalent to prove that $c_t > 0$ when $M_{t+1} = 0$. If M_{t+1} , then

$$c_{t+1} - c_t = F_t^{CA} + \frac{1}{b} \ln \left(\frac{\phi}{(1 - \rho_t)} \right) \leq 0$$

and

$$c_t \geq c_{t+1} = w_{t+1} + (1 - \rho_{t+1})M_{t+2} \geq 0$$

■

Therefore, most of the results derived in the case of a time-neutral consumer hold also for a impatient individual.

3.2 CRRA utility function

In this section I consider the case of an isoelastic per-period felicity:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{7}$$

where σ is the coefficient of relative risk aversion. The isoelastic utility function is usually preferred to the CARA specification because it implies a positive level of consumption.

Proposition 5 *With a CRRA utility function, the optimal refinancing policy is as fol-*

lows. Define:

$$\begin{aligned}
F_t^{CR} &= w_{t+1} + (1 - \rho_{t+1})M_{t+2} - \left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}} (w_t - M_t) \quad \text{for } t = 1 \dots T \quad (8) \\
&= w_T - \left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}} (w_{T-1} + M_{T-1}) - p_T \quad \text{for } t = T - 1 \quad (9)
\end{aligned}$$

Then:

$$M_{t+1} = \begin{cases} 0 & \text{if } F_t^{CR} < 0 \\ p_t & \text{if } F_t^{CR} > \left[\left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}} (1 - \rho_t) + 1\right] p_t \\ \frac{F_t^{CR}}{\left[\left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}} (1 - \rho_t) + 1\right]} & \text{otherwise} \end{cases}$$

Proof. From equation 2:

$$\begin{aligned}
\frac{\partial v_t}{\partial M_{t+1}} &= (1 - \rho_t) (w_t + (1 - \rho_t)M_{t+1} - M_t)^{-\sigma} \\
&\quad - \phi (w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1})^{-\sigma}
\end{aligned}$$

Hence, $M_{t+1} = 0$ if

$$\begin{aligned}
(1 - \rho_t) (w_t + (1 - \rho_t)p_t - M_t)^{-\sigma} \\
- \phi (w_{t+1} + (1 - \rho_{t+1})M_{t+2} - p_t)^{-\sigma} > 0
\end{aligned}$$

and $M_{t+1} = 0$ if

$$(1 - \rho_t)(w_t - M_t)^{-\sigma} \\ - \phi(w_{t+1} + (1 - \rho_{t+1})M_{t+2})^{-\sigma} < 0$$

Otherwise, there is an interior solution:

$$(1 - \rho_t)(w_t + (1 - \rho_t)M_{t+1} - M_t)^{-\sigma} \\ = \phi(w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1})^{-\sigma}$$

that implies

$$\left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}}(w_t + (1 - \rho_t)M_{t+1} - M_t) = w_{t+1} + (1 - \rho_{t+1})M_{t+2} - M_{t+1} \\ M_{t+1} = \frac{1}{\left[\left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}}(1 - \rho_t) + 1\right]} \left\{w_{t+1} + (1 - \rho_{t+1})M_{t+2} - \left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}}(w_t - M_t)\right\}$$

■

It should be noted that it is always sub-optimal to pay off the mortgage when $w_{t+1} > \left(\frac{\phi}{1 - \rho_t}\right)^{\frac{1}{\sigma}} w_t$. This result is the analogue of corollary 7 in the case of an isoelastic utility function.

4 The model with transaction costs

In this section I extend the previous model by allowing for a second source of imperfection in the mortgage market: refinancing the mortgage involves the payment of a fixed transaction cost k . Consumers enter each period with an outstanding debt M_t and they have to decide whether to refinance it or not. If they keep their existing mortgage, they have to pay to the lender the annual interest on their loan ($\rho_t M_t$). If they do refinance, they have to redeem the existing debt and take out a new mortgage $M_{t+1} \leq p_t$, again with maturity date T . The level of consumption c_t is determined by the budget constraint and w_t , ρ_t and p_t are time varying, but perfectly foreseen. Since saving and borrowing are precluded by assumption, individuals maximise life-time utility through their mortgage decision, subject to a budget constraint that depends on whether refinancing is undertaken or not.

In any period before the final one the general form of the budget constraint allowing for mortgage refinancing is:

$$c_t^R = w_t + (1 - \rho_t)M_{t+1} - M_t - k \quad (10)$$

Without refinancing, $M_{t+1} = M_t$ and $k = 0$, because no transaction costs have to be paid. Hence, the budget constraint takes the form:

$$c_t^{NR} = w_t - \rho_t M_t \quad (11)$$

Let ϕ be the intertemporal discount factor and $u(c_t)$ the per-period utility function. As

in Angelini and Simmons (2005), in any $t < T$ the value function is the maximum of the value function with refinancing and the value function without any mortgage refinancing:

$$V_t = \max(V_t^R, V_t^{NR}) \quad (12)$$

where

$$V_t^R = \max_{M_{t+1}} [u(c_t^R) + \phi V_{t+1}]$$

and

$$V_t^{NR} = u(c_t^{NR}) + \phi V_{t+1}$$

The presence of a transaction cost adds complexity to the problem and the optimal refinancing policy becomes much less transparent. Therefore, in what follows I will present the analytical solution of the problem only in the case of a simple three period model. The solution is obtained by backward induction.

4.1 T

As in the previous model, in the last period the house is sold and any outstanding mortgage is redeemed, so that there is no issue of refinancing for period $T + 1$, but now the consumer has to pay a fixed transaction cost if she enters the final period with a positive debt:

$$c_T^R = c_T^{NR} = c_T = w_T + p_T - M_T - k_T$$

where $k_T = k$ if $M_T > 0$ and $k_T = 0$ if $M_T = 0$. The value function takes the form:

$$V_T = V_T^R = V_T^{NR} = u(c_T) = u(w_T + p_T - M_T - k_T) \quad (13)$$

4.2 T-1

In period $T-1$, if no refinancing is undertaken, $M_T = M_{T-1}$ and the budget constraint is:

$$c_{T-1}^{NR} = w_{T-1} - \rho_{T-1}M_{T-1}$$

In this case the value function is simply:

$$V_{T-1}^{NR} = u(c_{T-1}^{NR}) + \phi V_T = u(w_{T-1} - \rho_{T-1}M_{T-1}) + \phi u(w_T + p_T - M_{T-1} - k_T) \quad (14)$$

On the contrary, if the consumer chooses to refinance, the budget constraint is:

$$c_{T-1} = w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1} - k$$

and the value function is given by:

$$\begin{aligned} V_{T-1}^R &= \max_{M_T} v_{T-1} = \max_{M_T} [u(c_{T-1}^R) + \phi V_T] \\ &= \max_{M_T} [u(w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1} - k) \\ &\quad + \phi u(w_T + p_T - M_T - k_T)] \end{aligned} \quad (15)$$

The derivative of v_{T-1} with respect to M_T is:

$$\frac{\partial v_{T-1}}{\partial M_T} = (1 - \rho_{T-1})u'(w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1} - k) - \phi u'(w_T + p_T - M_T - k_T)$$

where the consumer redeems any outstanding debt if

$$(1 - \rho_{T-1})u'(w_{T-1} - M_{T-1} - k) - \phi u'(w_T + p_T) < 0$$

she refinances to the maximum possible extent if

$$(1 - \rho_{T-1})u'(w_{T-1} + (1 - \rho_{T-1})p_{T-1} - M_{T-1} - k) - \phi u'(w_T + p_T - p_{T-1} - k) > 0$$

and she choose a mortgage size that solves

$$(1 - \rho_{T-1})u'(w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1} - k) = \phi u'(w_T + p_T - M_T - k)$$

otherwise. If the consumer is time neutral (i.e. $\phi = 1 - \rho_{T-1}$) or if the utility function is separable in k , such as the CARA felicity, then the interior solution for the mortgage takes the same form as in the case with no transaction costs. It should also be noted that in the period before the final one, the individual might have an incentive to redeem the mortgage in order to be exempted from the payment of the transaction cost at T .

Remark 6 *At $T - 1$ the transaction cost not only determines whether consumers refinance or not, but it has also an influence on the extent to which they do so.*

Consumers choose not to refinance when $V_{T-1}^{NR} > V_{T-1}^R$, i.e. if:

$$\begin{aligned} & u(w_{T-1} - \rho_{T-1}M_{T-1}) + \phi u(w_T + p_T - M_{T-1} - k_T) \\ & > u(w_{T-1} + (1 - \rho_{T-1})M_T - M_{T-1} - k) + \phi u(w_T + p_T - M_T - k_T) \end{aligned} \quad (16)$$

where M_T is the optimal mortgage size. It is noteworthy that the inequality always holds if $M_T > M_{T-1} > 0$ and

$$(1 - \rho_{T-1})(M_T - M_{T-1}) < k$$

Remark 7 *The consumer does not refinance when the value of the equity that is optimal to release from the house is lower than the transaction cost that should be paid.*

Furthermore, if the optimal mortgage conditional on refinancing is such that $M_T = M_{T-1}$, then by definition $V_{T-1}^{NR} > V_{T-1}^R$.

4.3 T-2

The refinancing decision in period $T - 2$ is conditional on the mortgage choice at $T - 1$ and on M_{T-2} , whose value can be treated as given in the case of a three period model. Two major cases can be identified, depending on the refinancing policy at $T - 1$.

Case 1

Suppose that in $T - 1$ for the consumer it will be optimal to refinance ($V_{T-1} = V_{T-1}^R$). Then, the value functions at $T - 2$ without and with refinancing respectively,

conditional on refinancing at $T - 1$, are given by the following expressions:

$$\begin{aligned}
V_{T-2}^{NR} &= u(c_{T-2}^{NR}) + \phi V_{T-1} \\
&= u(w_{T-2} - \rho_{T-2} M_{T-2}) \\
&\quad + \phi \max_{M_T} [u(w_{T-1} + (1 - \rho_{T-1}) M_T - M_{T-2} - k) \\
&\quad + \phi u(w_T + p_T - M_T - k_T)]
\end{aligned} \tag{17}$$

$$\begin{aligned}
V_{T-2}^R &= \max_{M_{T-1}} v_{T-2} = \max_{M_{T-1}} [u(c_{T-2}^R) + \phi V_{T-1}] \\
&= \max_{M_{T-1}} \{u(w_{T-2} + (1 - \rho_{T-2}) M_{T-1} - M_{T-2} - k) \\
&\quad + \phi \max_{M_T} [u(w_{T-1} + (1 - \rho_{T-1}) M_T - M_{T-2} - k) \\
&\quad + \phi u(w_T + p_T - M_T - k_T)]\}
\end{aligned} \tag{18}$$

Since k is a constant, if the individual chooses to refinance in both period T and $T - 1$, the optimal mortgage size is determined as in the case of no transaction costs (see section 3). Consumers do not refinance if $V_{T-2}^{NR} > V_{T-2}^R$.

Case 2

Consider now the case where for the consumer it will not be optimal to refinance in $T - 1$ ($V_{T-1} = V_{T-1}^{NR}$). Then, the value functions at $T - 2$ without and with refinancing

respectively are given by the following expressions:

$$V_{T-2}^{NR} = u(c_{T-2}^{NR}) + \phi V_{T-1} \quad (19)$$

$$= u(w_{T-2} - \rho_{T-2} M_{T-2}) \\ + \phi[u(w_{T-1} - \rho_{T-1} M_{T-2}) + \phi u(w_T + p_T - M_{T-2} - k_T)]$$

$$V_{T-2}^R = \max_{M_{T-1}} \{u(w_{T-2} + (1 - \rho_{T-2}) M_{T-1} - M_{T-2} - k) \quad (20)$$

$$+ \phi[u(w_{T-1} - \rho_{T-1} M_{T-1}) + \phi u(w_T + p_T - M_{T-1} - k_T)]\}$$

It should be noted that if $M_{T-1} > M_{T-2}$, then the consumer will not refinance if:

$$(1 - \rho_{T-2})(M_{T-1} - M_{T-2}) < k$$

This is the analogue of remark 11 for period $T - 2$. To determine the optimal mortgage size conditional on refinancing, I have to evaluate the sign of the first derivative of the value function with respect to M_{T-1} :

$$G(M_{T-1}) = (1 - \rho_{T-2})u'(w_{T-2} + (1 - \rho_{T-2})M_{T-1} - M_{T-2} - k) \\ - \rho_{T-1}\phi u'(w_{T-1} - \rho_{T-1}M_{T-1}) - \phi^2 u'(M_{T-1} + k - w_T + p_T)$$

Therefore $M_{T-1} = 0$ if $G(M_{T-1} = 0) < 0$; $M_{T-1} = p_{T-2}$ if $G(M_{T-1} = p_{T-2}) > 0$; otherwise there is an interior mortgage that solves $G = 0$.

4.4 Some general results

The aim of this section is to extend some of the results obtained for a simple three period model to a more general case in which the individual lives for T periods. For simplicity, I assume that the utility function is isoelastic and takes the form in (7).

First, suppose that at t the consumer chooses to pay off the existing debt and take out a new mortgage, which she will hold until its maturity date T . Hence, the individual maximisation problem becomes:

$$\max_{M_{t+1}} \left\{ \frac{(w_t + (1 - \rho_t)M_{t+1} - M_t - k)^{1-\sigma}}{1 - \sigma} + \sum_{s=t+1}^{T-1} \phi^{s-t} \frac{(w_s - \rho_s M_{t+1})^{1-\sigma}}{1 - \sigma} + \phi^{T-t} \frac{(w_T + p_T - M_{t+1} - k_T)^{1-\sigma}}{1 - \sigma} \right\}$$

and the first order condition is:

$$(1 - \rho_t)(w_t + (1 - \rho_t)M_{t+1} - M_t - k)^{-\sigma} - \sum_{s=t+1}^{T-1} \phi^{s-t} \rho_s (w_s - \rho_s M_{t+1})^{-\sigma} - \phi^{T-t} (w_T + p_T - M_{t+1} - k_T)^{-\sigma}$$

Not surprisingly, the higher the number of periods in which the consumer will not refinance, the lower the amount of equity that it is optimal to withdraw from the house in the current period. In fact, to take out a maximum mortgage has a positive effect in the short-run (current consumption jumps), but a negative effect in all future periods in which the individual will have to pay to the lender the annual interest on the loan.

Furthermore, as stated in remark 11, the consumer does not refinance the mortgage when the amount of equity that it would be optimal to release from the house is lower

than the cost that should be paid to do so. In other words, the optimal refinancing policy follows an (S,s) rule, characterised by infrequent adjustments. In fact, if the transaction cost is high, consumers refinance only when the optimal value of the mortgage substantially departs from the actual current value. On the contrary, if the optimal and current values are not remarkably different, agents choose to refinance solely if this involves the payment of a negligible transaction cost. In the limit case where $k = 0$, individuals adjust to the optimal stock of debt immediately and the optimal mortgage policy is the one described in section 3.

5 Calibrated simulations

The closed-form solution derived in this paper provides an easy way to evaluate the effects of changing parametric values on the optimal refinancing decision. In this section I present some calibrated simulations of the optimal life-time mortgage choice of an individual who lives for 40 periods ($T = 40$) and whose only asset in the portfolio is the house where she lives. I run the simulations for the case of an isoelastic utility function with $\sigma = 0.2$. Annual wages are hump-shaped and are defined by:

$$w_t = 0.5 \cdot \exp \left[- \left(\frac{t - 20}{20} \right)^2 \right]$$

I also assume that in the initial period the mortgage is equal to the house price, with $p_1 = 1.75 \cdot w_1$.

In the first simulation I assume a growing profile of house prices. In figure 3 and 4 I plot labour income, house prices, the mortgage rate and the optimal mortgage choice

of two individuals who differ only with respect to their rate of time preference. In the first part of their life, when both wages and house prices are expected to be growing, agents use mortgage equity withdrawal to finance their consumption needs: individuals can increase their outstanding debt in every period because they know that they will be able to pay the interest rates on the new mortgage (wages are growing). However, the more patient individual (figure 3) slightly decreases the level of the loan in periods in which the mortgage rate is high, while the other chooses to refinance to the maximum possible extent in every period. In the second half of their life, both individuals tend to decrease or increase their debt position depending on the mortgage rate, but they differ with respect to the amount of equity that they inject or withdraw from the house: the level of indebtedness of the less patient individual (figure 4) is always higher than that of the other consumer. Towards the end of life consumers increase their mortgage in every period to finance consumption because they know that at T the house will be sold at a high price and they will be able to pay off the debt.

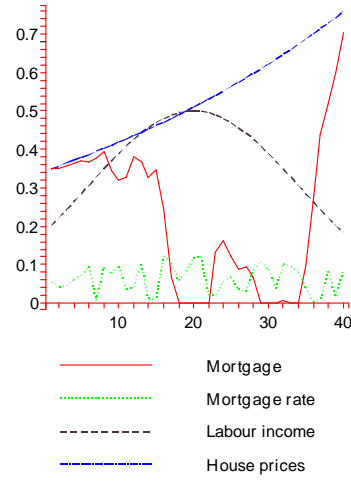


Figure 3 $\phi = 0.94$

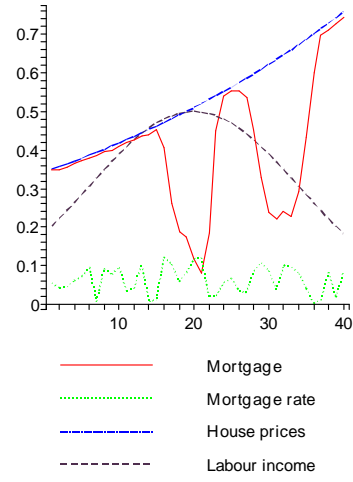


Figure 4 $\phi = 0.92$

In the second simulation house prices fluctuate over time and are given by:

$$p_{t+1} = p_t + \varepsilon_t \quad \text{where } \varepsilon_t \sim U(-0.1, 0.1) \text{ and } p_1 = 1.75 \cdot w_1$$

All other assumptions are maintained.



Figure 5 $\phi = 0.94$

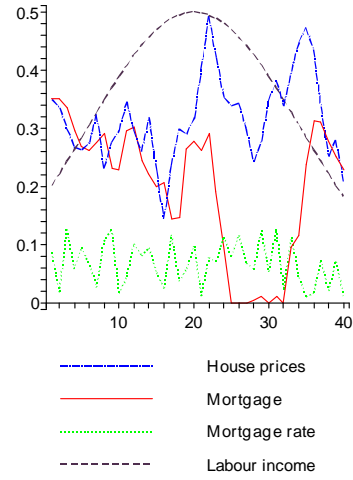


Figure 6. $\phi = 0.92$

When house prices fluctuate over time and wages are growing, the more impatient individual, whose mortgage choice is represented in figure 6, never pays off the outstanding debt, even in periods in which the mortgage rate (i.e. the cost of the loan) is particularly high. However, she is forced to redeem the mortgage when wages start to decrease because otherwise she would not be able to pay to the lender the annual interest on the loan. Only towards the end of life this more impatient consumer can increase her debt position, which she will repay in the last period with the proceedings given by the sale of the house. Figure 5 shows the mortgage choice of a more patient individual who prefers to resort less to mortgage refinancing because she attaches a greater value to the future and she knows that to release a large amount of equity from the house today implies having to pay high interests on the loan in all future periods.

Note that in both simulations consumers are more likely to withdraw a large amount

of equity from their properties when both house prices and wages are expected to be growing. Certainly, the extent to which they do so depends also on other parameters, such as the mortgage rate and the rate of time preference.

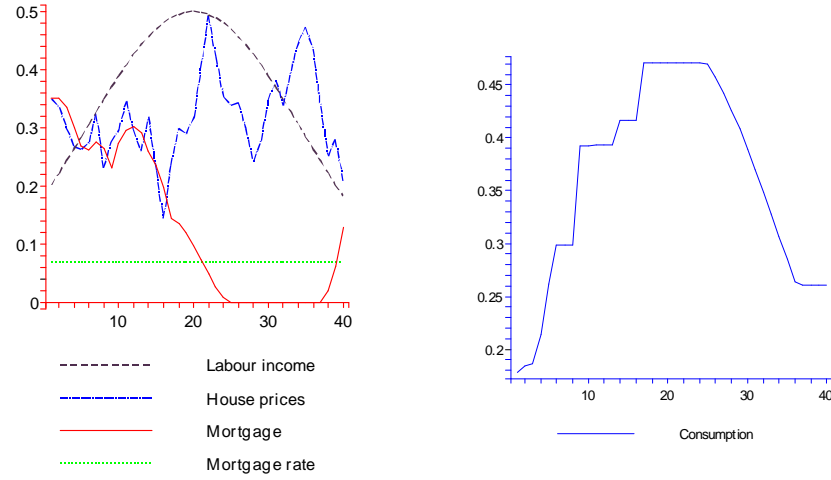


Figure 7. Time neutral consumer.

Figure 7 represents the optimal refinancing policy and life-time consumption of a time neutral individual with $\phi = 1 - \rho = 0.93$. It is noteworthy that in this case the consumer chooses a bang-bang solution (either a 0 or a 100% mortgage) in most periods of her life. In such periods, even if the individual is time-neutral and she would like to smooth consumption over time, she is prevented from doing so by the upper and lower bounds on the amount that can be borrowed.

6 Conclusions

The percentage of homeowners in the UK is about 70 percent. Consumers hold a large fraction of their wealth in housing, but they do not trade frequently because of high pecuniary and non-pecuniary costs. However, home equity can act as a financial buffer: through remortgaging, investors can release equity from the value of their home either to invest in other assets in order to adjust their portfolio or for consumption smoothing purposes. In this paper I have analysed the optimal mortgage refinancing policy of a homeowner who cannot save and who can borrow only using the house as collateral for secured loans. Since in the model there are no other assets but the house, the only reason why investors remortgage is to smooth consumption. The results of the model show that, when current income is low with respect to future income, agents respond by withdrawing equity from the house to smooth consumption. Therefore, it might be optimal to refinance even in a world of stable or rising interest rates.

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A Appendix

A.1 Proof of proposition 1

$$\begin{aligned} \frac{\partial \nu_t}{\partial M_{t+1}} = & \frac{\partial u(M_t, M_{t+1})}{\partial M_{t+1}} + \phi \frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+1}} + \\ & \phi \left[\frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+2}} \frac{\partial M_{t+2}}{\partial M_{t+1}} + \phi \frac{\partial V_{t+1}(M_{t+2})}{\partial M_{t+2}} \frac{\partial M_{t+2}}{\partial M_{t+1}} \right] \end{aligned}$$

In what follows I want to prove that the term in parentheses is equal to 0. Let us start by considering the optimisation problem that the consumer faces at period $t + 1$.

$$V_{t+1} = \max_{M_{t+2}} [u(M_{t+1}, M_{t+2}) + \phi V_{t+2}]$$

Thus, $M_{t+2} = p_{t+1}$ if

$$\left(\frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+2}} + \phi \frac{\partial V_{t+2}}{\partial M_{t+2}} \right) \Big|_{M_{t+2}=p_{t+1}} > 0$$

$M_{t+2} = 0$ if

$$\left(\frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+2}} + \phi \frac{\partial V_{t+2}}{\partial M_{t+2}} \right) \Big|_{M_{t+2}=p_{t+1}} < 0$$

Otherwise M_{t+2} is an interior solution and solves:

$$\frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+2}} + \phi \frac{\partial V_{t+2}}{\partial M_{t+2}} = 0$$

Consider now the term $\frac{\partial M_{t+2}}{\partial M_{t+1}}$, where $M_{t+2} \in [0, p_{t+1}]$ and suppose that the size of the mortgage at $t + 2$ depends on the size of the mortgage at $t + 1$.

$$M_{t+2} = \begin{cases} 0 & \text{if } M_{t+1} \in A \\ f(M_{t+1}) & \text{if } M_{t+1} \in B \\ p_{t+1} & \text{if } M_{t+1} \in C \end{cases}$$

It follows that when M_{t+2} is a corner solution, $\frac{\partial M_{t+2}}{\partial M_{t+1}} = 0$:

$$\frac{\partial M_{t+2}}{\partial M_{t+1}} = \begin{cases} 0 & \text{if } M_{t+1} \in A \\ f'(M_{t+1}) & \text{if } M_{t+1} \in B \\ 0 & \text{if } M_{t+1} \in C \end{cases}$$

Therefore:

$$\left[\frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+2}} + \phi \frac{\partial V_{t+2}}{\partial M_{t+2}} \right] \frac{\partial M_{t+2}}{\partial M_{t+1}} = 0$$

and

$$\begin{aligned} \frac{\partial \nu_t}{\partial M_{t+1}} &= \frac{\partial u(M_t, M_{t+1})}{\partial M_{t+1}} + \phi \frac{\partial u(M_{t+1}, M_{t+2})}{\partial M_{t+1}} \\ &= u'(c_t) \frac{\partial c_t}{\partial M_{t+1}} + \phi u'(c_{t+1}) \frac{\partial c_{t+1}}{\partial M_{t+1}} \\ &= u'(c_t)(1 - \rho_t) - \phi u'(c_{t+1}) \end{aligned}$$

that proves proposition 1.